Open Mathematical Olympiad

Grade 5

5.1. Write numbers from 1 to 12 in the circles in Figure 1, each circle containing a different number, such that the sum of the three numbers on each side of the hexagon is the same. It is sufficient to provide a single example showing how to do this.

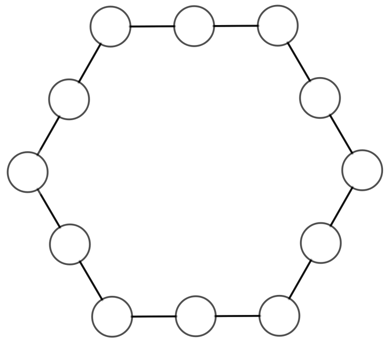

Figure 1

5.2. Fill in digits in place of the letters and asterisks so that it forms a correct multiplication example. The same letter must represent the same digit, but different letters must represent different digits. Asterisks can represent any digits, including those that may be used in place of letters. None of the digits (neither in place of letters nor asterisks) may be 0. The letters "a" and "ā" represent different digits. It is sufficient to provide a single example showing how to do this.

	×	m	a
		t	e
	*	*	*
*	m	ā	
t	i	k	a

5.3. What is the smallest possible sum of digits of a six-digit number that is divisible by 24?

5.4. Show how a 5×7 rectangular grid can be cut into one 1×3 rectangular grid (Figure 2) and eight "T" shaped figures (Figure 3); both types of figures may also be rotated.

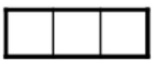

Figure 2

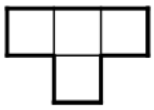

Figure 3

5.5. Initially, in a 3×3 grid table, zero is written in each cell. In one move, Raitis chooses any 2×2 grid square and adds 1 to all four numbers in it. After several such moves, he carefully examined the resulting 9 numbers. Can it be that exactly 8 of them are prime numbers? Can it be that all 9 are prime numbers?

Open Mathematical Olympiad

Grade 6

6.1. Write numbers from 1 to 14 in the circles in Figure 1, each circle containing a different number, such that the sum of the three numbers on each side of the heptagon is the same. It is sufficient to provide a single example showing how to do this.

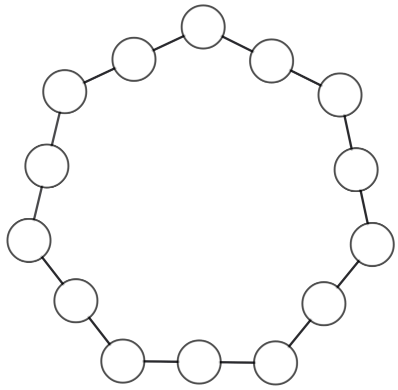

Figure 1

6.2. Show how a 9×11 grid rectangle can be cut into one 1×3 grid rectangle (Figure 2) and 24 ”T” shaped figures (Figure 3). Both types of figures may also be rotated.

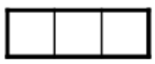

Figure 2

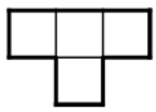

Figure 3

6.3. Fill in digits in place of the letters to make a correct addition example, where the same letter represents the same digit, but different letters represent different digits. The letters “a” and “ā” represent different digits. It is sufficient to provide a single example showing how to do this. Remember that a number cannot start with zero!

$$\begin{array}{ccccccc}
 & \mathrm{p} & \mathrm{a} & \mathrm{s} & \mathrm{a} & \mathrm{k} & \mathrm{a} \\
+ & & \mathrm{a} & \mathrm{s} & \mathrm{a} & \mathrm{k} & \mathrm{a} \\
+ & & & \mathrm{s} & \mathrm{a} & \mathrm{k} & \mathrm{a} \\
+ & & & & \mathrm{a} & \mathrm{k} & \mathrm{a} \\
+ & & & & & \mathrm{k} & \mathrm{a} \\
+ & & & & & & \mathrm{a} \\
\hline
\mathrm{g} & \mathrm{r} & \bar{\mathrm{a}} & \mathrm{m} & \mathrm{a} & \mathrm{t} & \mathrm{a}
\end{array}$$

6.4. Professor Cipariņš wrote two seven-digit numbers on the board. He replaced each digit in these numbers with a letter, where the same letter represents the same digit and different letters represent different digits. It is known that one of the written numbers PLATONS is divisible by 45. Prove that the other written number SOKRATS is not divisible by 54.

6.5. In 2786, the Mars Central Post sent exactly 10% more telegrams than in 2785. In each of the following three years (2787, 2788, and 2789), they sent exactly 10% more telegrams than the previous year. If no more than 15,000 telegrams were sent in 2785, how many telegrams were sent from the Mars Central Post in 2789?

Open Mathematical Olympiad

Grade 7

7.1. Five houses stand in a row on one side of a street. Each house has at least two residents, and no two houses have the same number of residents.
A person's neighbors are defined as all people who live either in the same house or in an adjacent house (not including the person themselves). Is it possible for each person to have either exactly 20 or exactly 30 neighbors?

7.2. Show how to cut a 3×3 grid square into 3 different pieces (cutting along grid lines) such that any pair of these pieces forms a symmetric figure when placed together in their original positions. Each piece must touch at least one edge of the square (a piece cannot consist only of the central cell).
A figure is symmetric if it can be folded in half so that both halves coincide. Two pieces are considered different if one cannot be transformed into the other by sliding or flipping.

7.3. What is the smallest possible sum of digits of a six-digit number that is divisible by 99?

7.4. Twelve glasses of lemonade are initially placed in a row on a table. Indriķis and Oto take turns, with Indriķis going first. On each turn, a player must drink either one glass or two adjacent glasses (the empty glasses remain on the table in their positions). The player who drinks the last glass empty wins. Which player can always guarantee a win with perfect play?

7.5. In 3546, the Saturn Space Station launched exactly 5% more interplanetary flights than in 3545. The number of flights continued to increase each year from 3547 to 3550. Table 1 shows the exact percentage increase over the previous year for each of these five years (3546 through 3550). If no more than one million flights launched from the Saturn Space Station in 3545, how many flights launched in 3550?

Year	Increase over previous year
3546	5%
3547	5%
3548	10%
3549	10%
3550	5%

Table 1

Open Mathematical Olympiad

Grade 8

8.1. Show how to express the number 2025 using exactly 9 ones, using arithmetic operations, exponentiation, and parentheses as needed. You may also place ones adjacent to each other without any operation symbol to form numbers like 11, 111, etc. If you cannot do this with 9 ones, show how to do it with as few ones as possible.

8.2. On the legs AB and BC of an isosceles triangle ABC, points M and K are chosen respectively such that segments AK and MC are perpendicular and $AM = AK = AC$. Find all angles of the triangle ABC.

8.3. In an infinite sequence of positive integers, each term starting from the second is obtained by adding the largest digit of the previous term to that term. For example, if the first term is 13, then the sequence is $13, 16, 22, 24, 28, 36, \ldots$. Does there exist a value for the first term such that all terms in the sequence are **a)** even numbers; **b)** odd numbers?

8.4. Ilmārs wants to carry 150 kg of pumpkins up to his apartment. Each pumpkin weighs at most 10 kg. In one trip, he can carry any collection of pumpkins with a total weight of at most 30 kg. What is the minimum number of trips that will guarantee Ilmārs can carry all the pumpkins, regardless of their individual weights? Cutting the pumpkins is not allowed!

8.5. LAPSA and SPALS are two five-digit numbers where digits have been replaced with letters: the same letter represents the same digit, and different letters represent different digits. Both numbers are divisible by 9. Prove that exactly one of them is divisible by 15. Which one is it?

Open Mathematical Olympiad

Grade 9

9.1. Find two different triples of positive integers (a, b, c) with $a < b < c$ such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{4}{2025}.$$

9.2. Point D is chosen on side AC of the triangle ABC such that the median AM of the triangle ABD is parallel to the median DN of the triangle DBC. Find the ratio $\frac{AD}{DC}$.

9.3. Does there exist a prime number p such that a $p \times p$ grid square can be cut along grid lines into smaller squares, each with side length at least 2?

9.4. In the infinite integer sequence $1, 2, 3, 6, 1, \ldots$, each term starting from the fourth is the last digit of the square of the sum of the previous three terms. Find the 2025th term of this sequence.

9.5. Ilmārs wants to carry 255 kg of pumpkins up to his apartment. Each pumpkin weighs at most 10 kg. In one trip, he can carry any collection of pumpkins with a total weight of at most 30 kg. What is the minimum number of trips that will guarantee Ilmārs can carry all the pumpkins, regardless of their individual weights? Cutting the pumpkins is not allowed!

Open Mathematical Olympiad

Grade 10

10.1. Given two different positive integers a and b, prove that

$$\frac{a}{b} + \frac{b}{a} \geq 2 + \frac{1}{ab}.$$

10.2. Squares $ABCD$ and $BCEF$ share the common side BC, and point M is the midpoint of the side EF. In what ratio does the line AC divide the segment MD?

10.3. In the infinite sequence of positive integers $24, 69, 21, 18, \ldots$, each term starting from the third is obtained by adding the digit sums of the two previous terms. Find the 2025th term of this sequence.

10.4. On a board, 2025 different positive integers are written. Each minute, Elīna chooses any two of them, say a and b, erases them, and writes the numbers $\gcd(a, b)$ and $\operatorname{lcm}(a, b)$ in their place. Prove that eventually (regardless of which numbers Elīna chooses), there will come a moment when she writes the same numbers on the board that she erased (in that same move).
Note: $\gcd(a, b)$ and $\operatorname{lcm}(a, b)$ denote the greatest common divisor and least common multiple of a and b, respectively.

10.5. Does there exist a prime number p such that a $p \times p$ grid square can be cut along grid lines into smaller squares where the side length (in grid units) of each smaller square is also a prime number?

Open Mathematical Olympiad

Grade 11

11.1. Prove that for any positive integer k, there exist positive integers a and b such that $a!$ is divisible by b, $b!$ is divisible by a, and $a - b = k$.
Note: The factorial of a positive integer n, denoted $n!$, is the product of all positive integers from 1 to n inclusive: $n! = 1 \cdot 2 \cdot \ldots \cdot n$. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.

11.2. The positive integers from 1 to 18 are written at the vertices and edge midpoints of a regular 9-gon (each number used exactly once) such that the sum of the three numbers on each edge is the same (each edge contains 3 numbers: two at vertices and one at its midpoint). What is the smallest possible value of this sum?

11.3. The diagonals of the convex quadrilateral $ABCD$ are perpendicular and intersect at point O. It is known that $BC = AO$. Point F is chosen such that $CF = BO$ and $CF \perp CD$. Prove that the triangle ADF is isosceles.

11.4. In how many different ways can the number 12 be expressed as a sum of ones, twos, and fours? Ways that differ only in the order of summands are considered different. For example, the number 4 can be expressed in six different ways: $1 + 1 + 1 + 1 = 1 + 1 + 2 = 1 + 2 + 1 = 2 + 1 + 1 = 2 + 2 = 4$.

11.5. Each of 7 frogs knows some verses from the frog anthem. It is known that any 3 frogs together can sing it completely (they collectively know all the verses). Can we guarantee that there exist two frogs who together can sing the entire frog anthem, if it has **a)** 20 verses; **b)** 21 verses?

Open Mathematical Olympiad

Grade 12

12.1. Find all real valued solutions to the equation

$$x \cdot \left(5^{\sqrt{x^2-1}} + 7^{\sqrt{x+1}}\right) = -2.$$

12.2. How many six-digit numbers are there that do not contain any of the digits $5, 6, 7, 8, 9, 0$ and have no two adjacent digits equal to 4?

12.3. In the triangle ABC, the altitude BD and the median BE are drawn. Point D lies between C and E, and $\sphericalangle ABE = \sphericalangle CBD = 23^\circ$. Find the measure of $\sphericalangle DBE$ in degrees!

12.4. Each of 7 frogs knows some verses from the frog anthem. Any 4 of them together can sing it completely (they collectively know all the verses). Can we guarantee that there exist three frogs who together can sing the entire frog anthem, if it has **a)** 34 verses; **b)** 35 verses?

12.5. Given positive integers x and y such that $x + 2y + 1$ is prime. Prove that $x^2 + 2xy - 2y$ cannot be a square number.